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TAYLOR-COUETTE FLOW AN EXPERIMENT ON VORTEX FLOW BETWEEN TWO CYLINDERS

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ABSTRACT

Taylor-Couette is the flow of an incompressible fluid in the gap between two concentric cylinders and is used for varying effects to examine turbulent patterns in fluids. The Couette apparatus was developed by Maurice Couette in 1890 as a means for measuring the viscosity of a fluid at small imposed angular velocities of the cylinders. Our apparatus has a stationary outer cylinder and an inner one rotating with variable speed. Above a certain angular speed the initial rotary laminar flow becomes unstable and organizes itself into a stack of toroidal vortices (Taylor vortices). By suitably dying the water, the apparatus allows for the visualization of these vortices and the observation of their dynamical evolution with changing speed. The aim of the experiment is to compare the Critical Taylor number (i.e., the point at which the Taylor vortices just starts to develop) measured experimentally with the theoretical value and also to find flow patterns in fluids.

Keywords: *Fluid Dynamics, Viscous Flow, Instability, Flow Visualization*

I. INTRODUCTION

Taylor-Couette flow is the unique outcome of a centrifugal instability resulting from the rotation of an inner cylinder relative to a concentric outer cylinder. Vortices of alternate senses circulate between the two cylinders and each vortex extends toroidally right round the annulus, so the overall flow is axisymmetric, these vortices are known as Taylor cells.

II. EQUATIONS OF FLUID MOTION

To solve for the velocity field of Taylor Couette flow before any instability is apparent, we must solve the Navier-Stokes equation for an incompressible, viscous fluid which is

$$\frac{\partial v}{\partial t} + (v.\nabla)v = -\frac{1}{\rho}\nabla p + \upsilon\nabla^2 v$$

and the continuity equation,

$$\nabla . v = 0$$

where, p is the pressure, υ is the kinematic viscosity and $v \equiv (v_r, v_\theta, v_z)$ is the velocity field.

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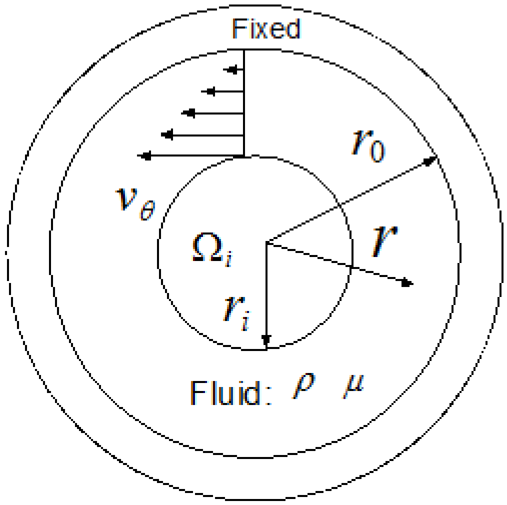

Fig. 1 Coordinate system for incompressible viscous flow between a fixed outer cylinder & a teadily rotating inner cylinder

Consider a fluid of constant density and viscosity between two concentric cylinders as shown in the Fig.1. There is no axial motion ($V_z = 0$) and let the inner cylinder rotate at angular velocity Ω_i. Let the outer cylinder be fixed. Using cylindrical coordinates throughout and because there is circular symmetry the velocity does not vary with θ and varies only with r.

The continuity equation can be written as,

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} = 0 = \frac{1}{r}\frac{d}{dr}(rv_r) \text{ or } rv_r = \text{Constant}$$

Since, $v_r = 0$ at both the inner and outer cylinders, it follows that $v_r = 0$ everywhere and the motion can only be purely circumferential,

$v_\theta = v_\theta(r)$. The θ - momentum equation becomes,

$$(V.\nabla)v_\theta + \frac{v_r v_\theta}{r} = -\frac{1}{r}\, r\frac{1}{\rho}\frac{\partial p}{\partial \theta} + g_\theta$$

$$+ \upsilon\left(\nabla^2 v_\theta - \frac{v_\theta}{r^2}\right)$$

For the conditions of the present problem, all the terms are zero except the last. Therefore the basic differential equation for flow between rotating cylinders is

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$$\nabla^2 v_\theta = \frac{1}{r}\frac{d}{dr}\left(r\frac{dv_\theta}{dr}\right) = \frac{v_\theta}{r^2}.$$

This is a linear second-order ordinary differential equation with the solution

$$v_\theta = C_1 r + \frac{C_2}{r}.$$

The constants are found by the no-slip condition at the inner and outer cylinders:

Outer, at $r = r_0$: $\quad v_\theta = 0 = C_1 r_0 + \dfrac{C_2}{r_0}$

Inner, at $r = r_i$: $\quad v_\theta = \Omega_i r_i = C_1 r_i + \dfrac{C_2}{r_i}.$

The velocity distribution solution can be written as:

$$v_\theta = \Omega_i r_i \frac{{r_0}/{r} - {r}/{r_0}}{{r_0}/{r_i} - {r_i}/{r_0}}.$$

III. TAYLOR-COUETTE INSTABILITY

Inviscid criterion problem was first considered by Rayleigh by neglecting viscous effects and discovered the source of instability for this problem and demonstrated a necessary and sufficient condition for instability that appears in the form of axisymmetric disturbances and this flow pattern, known as Taylor vortex flow is shown in Fig.2

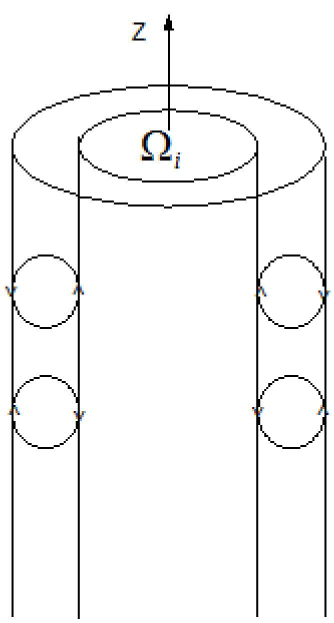

fig. 2 Taylor-Couette Instability

The nature of the instability producing Taylor cells can be explained by considering a toroidal element of fluid, i.e., in cylindrical polar coordinates, the fluid between r and r + dr and between z and z + dz. If it is displayed to a slightly larger radius and rotating faster, the radial pressure gradient coupled with the basic flow will be enough to balance the centrifugal force associated with the displaced element. The element then move still further outwards, also similarly an element displaced to a slightly lesser radius will move still further inwards. There will be an instability associated with some distributions of angular velocity.

When the two cylinders are rotating in the same sense the flow is either stable everywhere or unstable everywhere and Rayleigh criterion is:

$$\Omega_1 \ r_1^2 > \Omega_2 \ r_2^2$$

The Taylor number which represents the rate between the centrifugal force and the viscous drag force is defined as

$$T_a = 4 \left(\frac{\Omega_1 r_1^2 - \Omega_2 r_2^2}{r_2^2 - r_1^2} \right) \frac{\Omega_1 d^4}{\upsilon^2}$$

Where, $d = r_2 - r_1$

The critical value of the Taylor number to develop Taylor-Couette instability is 3416. The main aim of the experiment is using Taylor equation to compare with theoretical value.

The Reynolds number which represents the rate between the inertial forces and the viscous forces is defined as:

$$R_e = \frac{\Omega_1 r_1 (r_2 - r_1)}{\nu}$$

IV. DESCRIPTION OF THE APPARATUS

The apparatus consists of two cylinders made of transparent plastic material of which the outer cylinder is stationary and inner one is rotatable with variable speed. The outer cylinder has a diameter of 28.0 cm and inner cylinder of 20.0 cm diameter of length 111.20 cm. The gap size between outer and inner cylinder is 4.0 cm. The whole apparatus is mounted on a cylindrical platform. An electric motor is fixed inside the cylindrical platform in order to rotate the inner cylinder and is powered by a controlled DC voltage supply with range 0V-30V with a resolution of 0.1V. An outlet pipe is provided for letting the fluid out.

V. WORKING FLUIDS

The fluid used for conducting the Taylor-Couette experiment is water, mixing with Kalliroscope which is a rheoscopic fluid made out of thin anisotropic platelets that reflect light for visualization of vortices and the observation of their dynamical evolution with changing speed. Glycerol is used to increase the density and viscosity of the sample and also if required colour dye is used. The following fluid mixture content is used as shown in the Table 1.0.

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Table 1.0 Composition Of The Fluid Mixture

Fluid Mixture	Volume (lit.)	Density (kg/m3)	Mass (gms)	Volume Percen-tage (%)
Water	11.6	997	11.5652	58.99
Glycerine	7.456	1261	9402.016	37.91
Kalliroscope	0.5352	997	533.594	2.72
Stabilizer	0.073	997	72.781	0.37
Total	19.6642		21573.59	100

VI. EXPERIMENTAL METHOD

The apparatus is filled with the appropriate percentage mixture of water, Glycerol and Kalliroscope. Initially the inner cylinder which is attached by a shaft is operated with minimum speed available in the electric motor, later the speed is increased in order to see the Taylor-Couette flow patterns in fluids and also to know the Critical Taylor number at which the flow occurs.

VII. CONCLUDING REMARKS

The following discussions and conclusions are drawn:

1. The minimum voltage required to rotate the inner cylinder is 1.6 and angular speed is 0.104 rad/sec and it was observed that Taylor cells formed at this angular speed but not clearly visible. The kinematic viscosity of the Glycerine was 0.036 cm^2/sec. At 0.245 rad/sec fully developed Taylor cells formed and turbulent at 0.705 rad/sec as shown in the Figures 3, 4 and 5. The error value of angular velocity is + or - 0.017.
2. Observed 15 numbers of Taylor cells formed in a length of 59.0 cm at an angular speed of 0.626 rad/sec and the axial wave length is: 15 Taylor cells / 59.0 cm = 0.254 cm^{-1}. The size of the each Taylor cell is 3.93 cm which is nearly equal to the gap size between the cylinders i.e., 4.0cm. This value obtained agrees well with the theoretical expectation that the cells should be equivalent to the gap size.
3. It has taken approx. 20 min. to approach equilibrium in order to identify the Taylor cells. The instability development of the flow patterns are shown in Figures 3, 4 and 5.

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Fig.3 Flow patterns in rotating Couette flow in stable condition (Purely Azimuthal Flow)

Fig.4 Flow patterns in rotating Couette flow showing Taylor Cells

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Fig.5 Flow patterns in rotating Couette flow in Turbulent condition

4. Initially we added 17% of glycerol of the total solution, we can't able to find the angular speed for the Critical Taylor number, afterwards we increased the percentage of the glycerol content to 27.0, then also we faced the same situation. Finally, we increased the glycerol content to 38.0%, even though by increasing glycerol content from 17.0% to 38.0%, it was impossible to identify exact angular velocity for critical Taylor number. If the gap size between the cylinders is small, then we can able to observe the Critical Taylor number within small % of glycerol, but in our experiment the gap size is more and we cant able to modify the gap size, thats why we tried to increase the kinematic viscosity in order to locate the angular speed for the critical Taylor number, but we didn't succeeded.
5. While observing the patterns of the flow in Taylor-Couette apparatus, there are some errors in the apparatus construction. When the lid on the cylinder is not closed, we observed a small oscillation of the internal cylinder. Due to this effect, gap between the two cylinders is not maintaining constant. Also, when lid is closed, then we observed a small friction between the lid and cylinder. Due to this effect, the rotation of the cylinder is affected (i.e., the speed is reduced), but this problem is solved by calibration curves as shown in the Graphs 1, 2 and 3.

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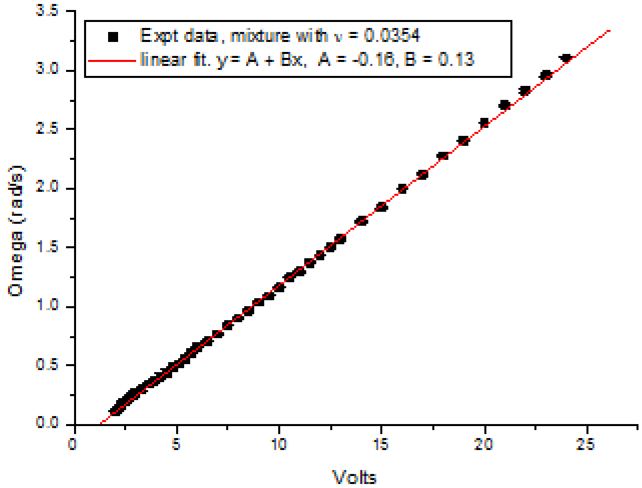
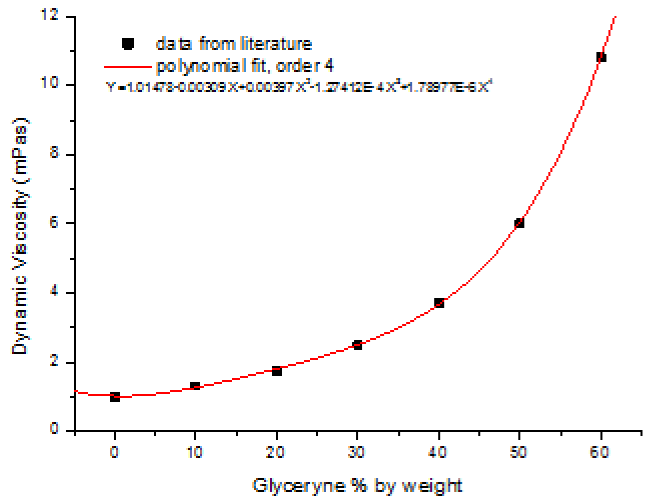
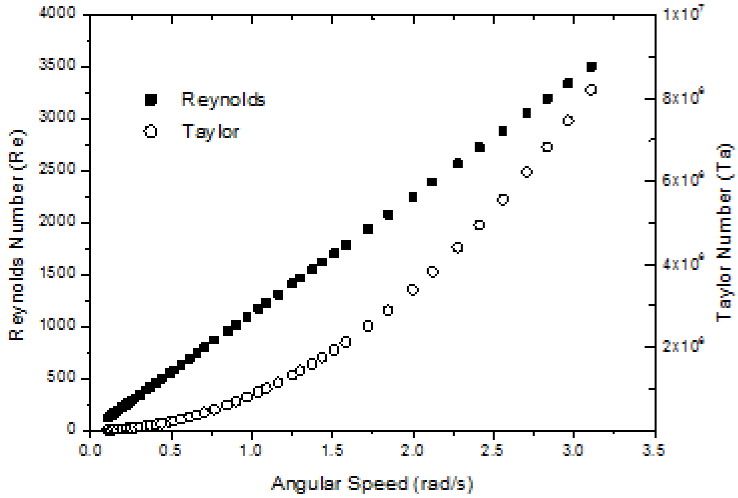


Graph 1



Graph 2



Graph 3

VIII. ACKNOWLEDGMENTS

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